Observational geometry (with simple reasoning). Formal geometry and beginning trigonometry. Trigonometry, with modern geometry.

The guiding principle is to begin the new subject always before the old one which leads to it has been completed.

Algebra is only arithmetic extended by the introduction of the following new ideas: 1. The use of letters to represent generalized number. 2. The use of the minus sign as a symbol of quantity. 3. The equation as a tool for the solution of problems. 4. The use of fractional and negative exponents. 5. The use of irrational numbers. 6. The use of the variable and its limit.

The sequence and manner of presenting these new ideas should be such that the motive for each shall be felt from the beginning. This can be done best by taking the equation as the organizing and developing idea for both matter and method in algebra.

Many of the operations of arithmetic must be restudied in algebra in the light of these new ideas.

The subject of factoring, which is so important and so troublesome to the learner, as also to the teacher, may be developed easily from three simple principles.

The theory of exponents and logarithms should be shown to rest upon these four elementary principles:

$$a^n \times a^m = a^{m+n};\ a^m/a^n = a^{m-n};\ (a)^{mn} = a^{mn},\ \text{and } (ab)^n = a^n b^n,$$

where ^{m} and ^{n} may be integral, fractional, positive, or negative. These principles should be carefully taught and their true relations to these theories clearly seen by the pupil.

The solution of the quadratic equation, and particularly of simultaneous equations depending upon quadratics, is very greatly aided by graphical methods.

Practical applications to second degree simultaneous equations arising out of practical problems.

Practical applications arising out of the geometry and trigonometry needed to render the selected area intelligible to grammar and high school pupils.

The formulas for stresses in beams, in columns, for flow of water, and for other subjects treated, in the strength of materials and hydraulics, furnish many instructive applications of algebraic principles, and should be freely drawn upon by the teacher.

Mathematics in the Primary Grades

Gertrude Van Hoesen

The chief purpose of the following part of the course in mathematics is to show the place and function of arithmetic in the primary grades. The outline presents various phases of work illustrating how essential an understanding of the fundamental principles and processes of arithmetic is in the most common affairs of life. Within the school, it is quite as essential that the child's work in arithmetic should be closely related to his life and interests. But it must be proved that such work is not only more interesting to the child, but also more educative.

The work in the outline below appears in an unrelated form, and is intended only for use as reference in the classroom discussion.

Discussion of methods of teaching numbers:

There are three ways—one prevailing, one in partial use, one which has never been fully carried out in any school.

The first method includes the notation and numeration of figures; adding, subtracting, multiplying, dividing; committing tables to memory; doing isolated problems; following a so-called logical order from adding to banking; learning rules and following them; learning definitions without understanding them.

In the second method blocks are used and other objects, such as shoepegs, sticks, and colored dishes; this includes counting, marking, and picturing. The objects have no interest beyond their mere use,

and the results have no value to the child. Yet the constant drill in comparison and the finding of the ratios of actual quantities give the child a certain power in mental imaging. This is a great improvement upon the mere memorizing of the names of figures, and the logical following of unpedagogical rules.

In the third method number is used for the legitimate purpose of measuring quantities that must be measured in order that conclusions sought by the pupils may be reached—not numbers for numbers' sake, but for their use.

It will be the endeavor, in this course of lectures, to use the third method, and to show that in order to teach science in its various branches—geography, which requires knowledge of distance, area, volume—history, in which dates, census records and the arts and industries of people are studied and compared—and manual training, which depends upon accurate measurements—it is absolutely necessary to teach number.

It is impossible to outline in this syllabus how number is used in teaching all of these subjects, but the following brief examples illustrate its use in geography, manual training, and physical training:

I. Geography.—This work will be used only when the results of such comparisons are of immediate practical use in geographical imaging.

Definite measuring in geography necessitates the use of — 1. Linear Measure.—The estimation and comparison of distances, length of coast line, length of rivers, height of mountains, depth of oceans. 2. Square Measure.—Area, comparative sizes of river-basins, continents, food areas, mining and forest areas. 3. Measure of Bulk and Volume.—Estimation and comparison of the amounts of material carried down by rivers in a day, month, or year. 4. Measure of Force and Energy.—Comparative temperatures and atmospheric pressures, etc. 5. Measurement of Time.—Estimation of distances in relation to standard rates of travel.

Climate.—All the facts gathered from the daily records concerning the temperature, direction of wind, rain and snow, length of day, etc., are valuable as data only as they arouse in the mind of the child an approximately accurate idea of local climatic conditions. Yet the constant accumulation and comparison of these facts bring about the use of all the fundamental operations of arithmetic, and give the necessary drill in the manipulation of figures. Through the interest in these observations, skill is acquired at the least expense of time and effort.

II. Manual Training.—The first step must be the working-drawing or plan of the object to be constructed; estimation of cost of suitable materials. The second step is the execution of the plan; for instance, the children need a pencil-box and decide that the dimensions may be 9x3x2 inches, and the material a certain kind of wood. The working-drawing consists of three rectangles; the first, 9x3 inches, represents the top and bottom; second, 9x2 inches, represents the sides; the third, 3x2 inches, represents the ends.

1. Compare length, breadth, and height. How many square inches of wood will actually be required to make the top and bottom? The sides? The ends? The entire box? If the board from which you cut the wood is 12 inches wide, how many inches of board will be needed when you reach the manual training room, allowing two inches between pieces? How many inches of board must necessarily be discarded? Knowing the cost of wood, estimate cost of material for box.

2. What theorems in geometry that the class can readily see are demonstrated in making this box? (a) From a point in a straight line one and only one perpendicular can be drawn; and (b) Converse of "a." (c) All right angles are equal. (d) Two straight lines perpendicular to the same straight line are parallel. (e) Converse of "d." (f) Two rectangles having equal bases and equal altitudes are equal. (g) Two rectangles having equal bases are to each other as their altitudes; *i. e.*, comparison of bottom and side. (h) Two rectangles having equal altitudes are to each other as their bases; *i. e.*, comparison of side and end.

NOTE.—As the articles needed become more difficult in construction, the problems become

proportionately difficult both in arithmetic and geometry.

III. Physical Culture.—Consideration of growth and exercise as a foundation for the study of hygienic and sanitary conditions.

Accurate physical measurements of children in October, January, and June. Height, weight, chest measurement, and chest capacity necessitate the use of measures of length, weight, and bulk. Compare the measurements of a normal child made by the Child Study Bureau. Chest measurement, preceded by study of properties of air, examination of exhaled air, test for carbonic acid gas. Problem: What is the average amount of air inhaled in one breath?

The new units are gills, pints, quarts, and gallons. Experiment: Use a gallon jar scaled on one side; invert over a pan of water and draw out as much air as possible in one inhalation through a rubber or bent-glass tube. What is the amount inhaled by each member of the class? Amount inhaled by the entire class? Average amount inhaled by each person in one inhalation? How long would it take to exhaust the air in the room if none of it were rebreathed? A bedroom is ten feet square and eleven feet high. How long would the enclosed air supply two persons? What would take place in the air if the room were unventilated?

Circulation: Count the number of heart beats and pulsations in wrist and temple per minute. Do they correspond? Find average for class. The blood is about 1-13 of the weight of the body (Foster). It has been estimated that at each systole the heart sends out 6.3 ounces of blood (Martin's *Human Body*). How much blood passes through the heart in one minute? One day? How many heart beats are necessary for all the blood in your body to pass through? Relation of pulsation to respiration: How many respirations in a minute? How much blood do you purify in one minute? What part of the air is oxygen? Then, what amount of oxygen do you use in one respiration?

All points will be illustrated by the work of the children in the various grades, as in the following outline, in which the function of arithmetic in the study of the home is shown.

But on account of limited space only those forms of mathematics in most common use in daily life will be suggested, and that in the briefest possible way.

IV. Equipment of the Home.—1. The number and value of the utensils necessary in the kitchen. 2. The number and value of the dishes necessary for proper service. 3. The amount and value of napery, tablecloths per yard. 4. Napkins per dozen. 5. Carpeting per yard. 6. Wallpaper per bolt. 7. Upholstery per yard. 8. Bed linen per yard. 9. Furniture.

Food.—(Note problems in chemistry and physics involved.) Hygienic dietary. Physiology of digestion. Relation of constituents of food to nourishment which they contain. Quantitative analysis of foods as to starch, albumen, carbon, fat, mineral matter, etc. Computation of the amount of the different food principles necessary for a normal child and adult. Comparison of amount of tissue-building materials with energy-producing materials. Inference.

Cooking.—Scientific Cooking.—Estimationof effects of temperature, water, various forms of bacteria, and the processes of digestion upon food constituents. Necessary uses of measures of weight—avoirdupois; of bulk—dry and liquid; of money—dollars, dimes. Means of measuring time and temperature. Success of practical cooks as dependent upon the unconscious accuracy of their judgments and the mathematical exactness of their measurements. Preparation of simple lunch, involving (a) a knowledge of food principles, (b) estimation of amount of articles selected, (c) estimation of the cost, (d) careful preparation of food in regard to food values and proportions.

Physics.—Incidental observation of evaporation and ebullition in cooking experiments. 1. Evaporation takes place at all temperatures, the amount varying with the exposed surface. 2. The rapidity of evaporation depends upon the kind of liquid. 3. Temperature of the boiling point remains constant. 4. The boiling point varies with the kind of liquid.

V. Farming.—The necessity for number in all farm work is illustrated in the following report; it represents a crop of corn grown on 9½ acres of land in Massachusetts:

1. Preparatory plowing and harrowing,	$ 5 00
2. Fertilizers; *i. e.*, 462 lb. of sulphate of ammonia, 177 lb. of muriate of potash and 163 lb. of bone-black treated with 81 lb. of sulphuric acid	33 00

3. Mixing, carting and spreading fertilizers	$ 1 40
4. Seeding and planting	1 60
5. Cultivation and hoeing	5 00
6. Cutting and stacking	3 00
7. Husking	11 50
8. Storing the fodder	7 50
9. Interest on land and taxes	5 50
Total expenses	$73 50
Besides the corn there was harvested 5.75 tons of fodder, to be sold at $8 per ton	$46 00
Actual cost of 115.5 bu. of corn	27 50
Actual cost of 1 bu. of corn	.238

When applied to the vast farms of the Middle States the problems increase. Especially important is the question of transportation there. (a) Railroads, surveying, etc. Continuous use of geometry and trigonometry is necessary. (b) Commerce—illustrated by duties: From the Daily News Almanac find the amount of money paid to the Government for imported goods. How has the Cuban war affected this? What articles which we use daily belong to the list? Give a list of problems which show the value of these articles before paying the duty; value afterward and value including commission to agents for buying them. (c) Census. The growth of the population shown by comparison of the first census with succeeding ones.

VI. Industries.—In the great industries and manufacturing plants and in our shops there is no problem too small to be of significance to the manager. Mathematical precision is at the basis of success. Number is involved in every move, from building and furnishing the plant, from the measurement and purchase of raw material, price of labor included, to the finished product.

Department of Art

John Duncan Antoinette B. Hollister Clara I. Mitchell

Three art courses will be given: Mr. Duncan, head of the department, will present a course in the principles of art, with illustrations. His work will begin July 15th, and continue for three weeks.

Miss Hollister's course will be clay modeling and painting, and Miss Mitchell's, industrial art with special relation to textile fabrics.

The design of these courses is to make modeling, painting, and drawing the means for self-expression and constructive activity, correlating them with all other subjects, and using them in the same way and for the same purpose that speech, oral reading, and writing should be used.

The child is recognized as a bundle of activities, eager to manifest himself in all possible ways.

Art as an outlet of personal energy satisfies normal desires and reacts upon growth and development.

The exercises will be simple and carefully adapted to elementary teaching; much of the work will be done in the field.

Principles of Art

JOHN DUNCAN

I. Art Teaching in Relation to Nature Study.

The course will begin with the simple and conventional methods which young children actually develop for themselves, and will thence progress through practical demonstrations in schoolroom and museum to open-air work in school garden, botanic and zoölogical garden, park and country.

As far as possible the study of nature will be completed by the correlated treatment of human occupations, thus leading to—

II. Figure Drawing and Composition.

Here will be indicated some methods of study and teaching taking the place of the usual work of a life class.

In both the above sections progressive exercises will be set in drawing, not only from direct observation, but from memory, and also from imagination.

III. Art in Home, School, and City Life.

Actual modes of decoration of living rooms, schoolrooms, etc. Possibilities of improvement in simple ways.

External aspect of buildings; as determined from within and from without; and with varying influence of material conditions and of ideals.

The changing aspect of cities, from age to